Journal of Engineering Physics and Thermophysics, Vol. 64, No. 6, 1993

ON THE EXPERIMENTAL DATA AND APPLIED MODELS OF TURBULENT HEAT TRANSFER IN NEAR-WALL FLOWS

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UDC 536.224

The data on the vector of the one-point correlation $\langle u_i t \rangle$ characterizing turbulent heat transfer are considered using the Reynolds approach to the turbulence description. The turbulent heat transfer from a given heat source (sink) is determined by the dynamic turbulence structure. With this taken into consideration, only the problem statements and obtained results are discussed in which both the thermal and dynamic characteristics of the liquid flow are discussed simultaneously using a unified approach.

At the Third Minsk Meeting (Belarus) on Heat and Mass Transfer in 1968 turbulent Prandtl numbers evaluated in air and mercury flows were reported [1]. Pr_T estimates obtained by different authors show random scatter of values from 0.3 to 1.5 and do not reveal even the qualitative character of Pr_T variation as a function of the distance from the wall and the Prandtl (air, Pr = 0.7; mercury, Pr ≈ 0.05) and Reynolds numbers. The sole experimental data presented on direct measurement of $\langle vt \rangle$ by a hot-wire anemometer in air illustrate, in fact, the same Pr_T scatter specified by low measurement precision and processing of primary results.

The review by A. Reynolds [2] in 1975 sums up the attempts to elaborate turbulent heat transfer (turbulent Prandtl number) models using the mixing length hypothesis. In the review, more than thirty calculation methods suggested in 1950-1975 are discussed, thus allowing the author to draw the conclusion that the considered models have added little to elucidation of the turbulent heat transfer mechanism in shear flows.

The present work is concerned, first of all, with the discussion of experimental data and their interrelation with modern turbulent heat transfer models. In Table 1 the problem under consideration, the main assumptions and designations, references to some published works and obtained results are schematically shown.

While in [2] the turbulent heat transfer models are based on the Prandtl and Kolmogorov turbulence hypotheses, at present they are most often related with the widespread two-parameter turbulence calculation model, i.e., with use of balance equations for two turbulence characteristics, namely, the kinetic turbulence energy and its dissipation intensity (the $k-\varepsilon$ model).

Some sufficiently justified propositions have been made.

In the case of developed equilibrium turbulent near-wall flow ($Re_x > 5 \cdot 10^6$, $Re > 2 \cdot 10^4$) the turbulent Prandtl number at Pr > 0.6 in the logarithmic-law region of the wall practically does not change with the distance from the wall, thus preserving a constant value in the range 0.85-0.91. In heat transfer calculations by the simplest models, with Pr_T = const being assumed over the entire section of the boundary layer, the above Pr_T values are used most often (see Table 1).

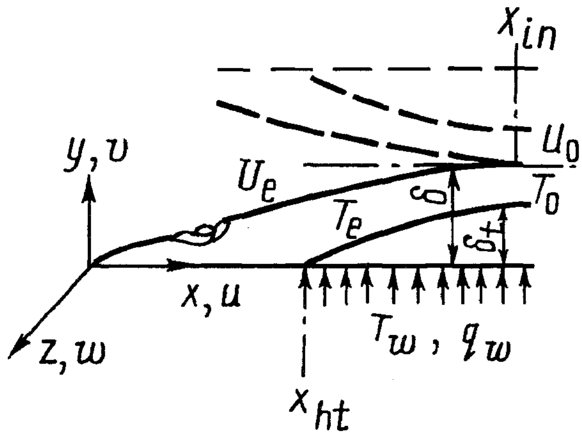
A vector of turbulent heat transfer is distinguished with strong anisotropy which takes account of shear turbulence anisotropy and may be essentially manifested provided thermal conditions are highly nonuniform. At the same time one must bear in mind that on representing it in the gradient form with respect to separate coordinates the turbulent thermal conductivities differ by orders of magnitudes ($\lambda_{Tx} \gg \lambda_{Tz} \gg \lambda_{Ty}$).

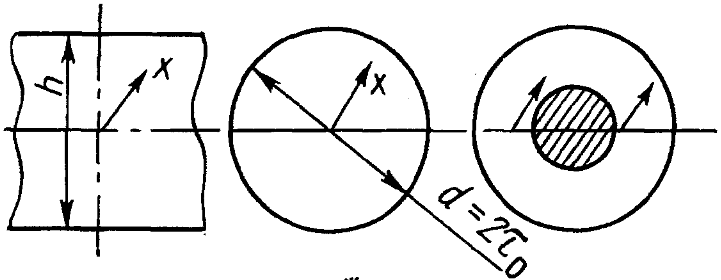
Depending on the position and intensity of heat sources (sinks), specific mechanisms of heat wave propagation and transfer may be essentially manifested (in particular, the effects of heat wave penetration into the wall flow and of temperature intermittence, see below).

Institute of High Temperatures, Russian Academy of Sciences, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 64, No. 6, pp. 689-697, June, 1993. Original article submitted May 21, 1992.

Table 1.

$$\langle u_i t \rangle$$





t – passive impurity

$$\beta = \frac{dP}{dx}\,\frac{\delta^*}{\tau_w} = \text{const}$$

(Clauser, 1959)

$$\delta^* = 2^n \int_0^{\delta} \left(1 - \frac{u}{u_e}\right)\left(\frac{r}{r_0}\right)^n dy \qquad (n = 0.1)$$

$$\langle wt \rangle = 0; \qquad \partial_y \langle vt \rangle \gg \partial_x \langle ut \rangle$$

$$\langle vt \rangle = (Pr_T)^{-1}\, \partial_y T\, (\langle uv \rangle / \partial_y U); \quad Pr_T = \frac{\nu_T}{a_T}$$

Pr_T = const

$$\left.\begin{array}{ll} 50 < y^+ < 300, & U^+ = \frac{1}{\mathcal{H}} \ln y^+ + B \\ Pr = 0.7, & T^+ = \frac{1}{\mathcal{H}_t} \ln y^+ + B_t \end{array}\right\} (\text{Э})$$

$$Pr_T = \frac{\mathcal{H}}{\mathcal{H}_t} = \frac{0.41}{\mathcal{H}_t} \longrightarrow 0.72;\ 0.85;\ \boxed{0.91}$$

(Gibson, 1988 [3])

$$St = \frac{C_f/2}{Pr_T + 11.0\sqrt{\frac{C_f}{2}}\,(Pr - Pr_T)} = St \quad (\text{Э})$$

(Ginzburg, 1970 [4])

$$\nu_T\left(y,\ Re;\ Tu,\ \frac{dP}{dx},\ \Delta,\ v_w\right)$$

Model

$$-0 \longrightarrow \nu_T = \left|\frac{dU}{dy}\right| l^2$$

(Prandtl)

$$-1\,(K) \longrightarrow \nu_T = C_1 K^{1/2} L$$

(Kolmogorov)

$$-2\,(K - E) \longrightarrow \nu_T = C_\mu K^2 / E$$

(Londer)

$\overline{Pr}_T$ (Re, Pr) – Fig.4

	Re	Pr
(Malhotra, 1988 [5])	$10^4 - 10^6$	$1 - 10^4$
(Myong, 1989 [6])	$2 \cdot 10^4 - 10^5$	$10^{-2} - 10^4$

$\overline{Pr}_T$(Tu) ≈ 0,6 [11]

$$Pr_T = Pr_{T0}\,[1 - 0.225\, th(0.2\,Tu)],$$

$$Pr = 0.7, \quad Pr_{T0} = 0.9 \quad [8]$$

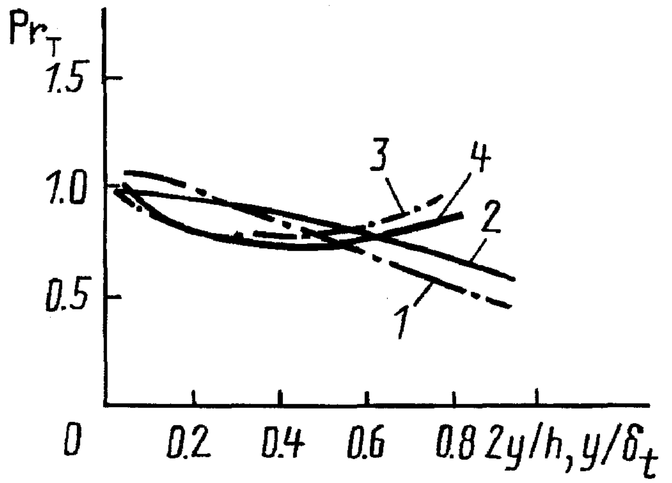
Pr_T (y)

(Pyadishyus, 1987 [8]) - Fig.3

(Hishida, 1986 [9]) - Fig.6

Pr_T (y, v_w, dP/dx)

(Elena, 1978 [12]) } - Fig.1
(Fulachier, 1978 [13]) }
(Roganov, 1984 [14]), [8] - Fig.2



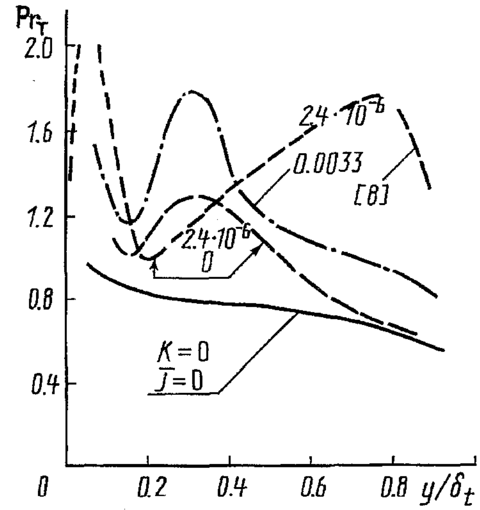


Fig. 1. Turbulent Prandtl number variation under injection conditions with an air flow in a channel [12] [1) $\bar{j} = (\rho V)_w/\overline{\rho U} = 0$; 2) 0.00165] and in a boundary layer [13] [3) $\bar{j} = (\rho V)_w/(\rho U)_\infty = 0$; 4) 0.003].

Fig. 2. Turbulent Prandtl number distribution with respect to the boundary layer thickness (Pr = 0.7) for a confuser flow and injection across a surface [14]; $K=(\nu/U_\infty^2)(dU_\infty/dx)$, $\bar{j}=0$, $K=2.25 \cdot 10^{-6}$ [8].

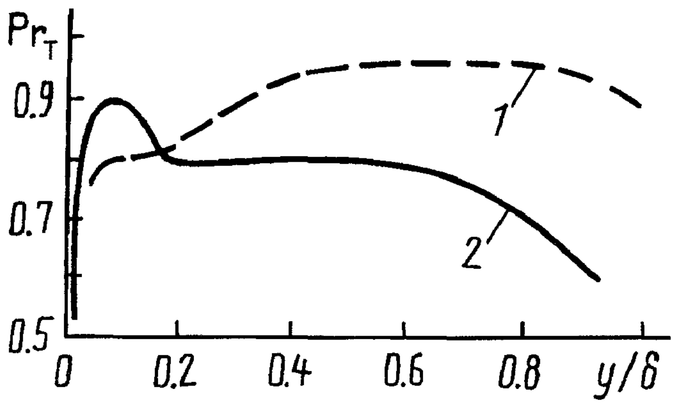
Experimental data show that when a turbulent air shear flow is exposed to different factors (a pressure gradient, external turbulence, surface roughness, mass forces, including a surface curvature, injection) the dependence $Pr_T(y)$ may become nonmonotonic and may have several extrema and its absolute value may attain values from 0.1 to very high ones, of the order of n*10.

It is established that in the developed-turbulence regions the turbulent Prandtl number is much lower as compared to the region with undeveloped (laminarized) turbulence, where it may considerably exceed unity.

Experimental data are obtained on statistical characteristics and the structure of turbulent heat transfer; some attempts are undertaken to simulate different conditions. However these results have not made a decisive contribution to understanding the physics of turbulent heat transfer in shear flows.

In the best experimental works concerned with investigation of the influence of any factor on heat transfer the turbulent heat transfer distributions have been measured preliminarily and by the same method but under conditions where the investigated factor exerts no influence. Such an experiment may be characterized as relative. Considering the nonlinearity of turbulent transfer processes, one would expect that the specific features of each experimental setup, bringing about its own uncontrollable disturbances, lead to appreciable differences in the turbulence characteristics if near-wall flows are subjected to the influence of the separate factors or their combinations employed in different works. As an example, we compare data presented in Figs. 1 and 2. It is established that flow acceleration characterized by the parameter $K=(\nu/U_\infty^2)(dU_\infty/dx)$ essentially influences the value of Pr_T and its distribution as shown in Fig. 2. Inspection of this figure also reveals a large divergence of the results obtained in different works practically at the same value $K = (2.25\text{-}2.4) \cdot 10^{-6}$. Also, it is shown that with application of a second factor (injection) the turbulent Prandtl number changes even more essentially. At the same time experimental data [12, 13] (see Fig. 1) obtained both for a boundary layer and for a tube only under injection conditions reveal no appreciable Pr_T variation. This is also typical for relative mass injection of air into a boundary layer practically of the same value, as in [14] (Fig. 2), i.e., $\bar{j} = (3\text{-}3.3) \cdot 10^{-3}$.

At present there are not even adequate physical considerations concerning the character of change of the available experimental data. This equilibrium boundary layer (represented in one chapter of the monograph), which are compared in Fig. 3 with the curve based on general physical considerations of the authors (given in another chapter).



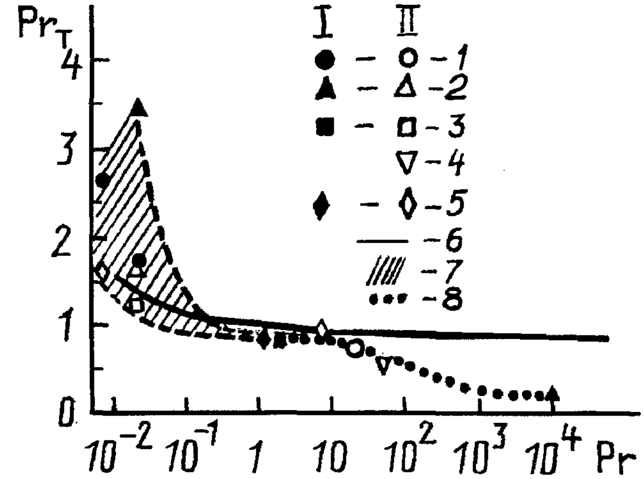


Fig. 3. Turbulent Prandtl number vs the distance from a wall in a boundary layer for Pr = 0.7 according to [8]: 1) Fig. 3.14 (physical model); 2) Fig. 5.10 (measurement results).

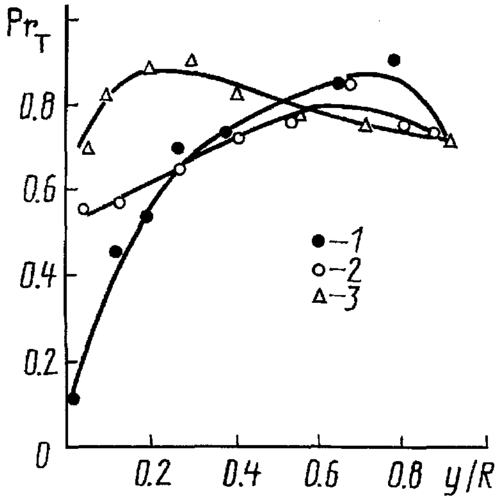
Fig. 4. Mean turbulent Prandtl number versus molecular Prandtl number for a tube flow [6]: I) $Re=2\cdot10^4$; II) 10^5; 1) Fuchs-Faesch; 2) Buhr et al.; 3) Subbatin et al.; 4) Sheriff-O.Kanl; 5) Smith et al.; 6) present model [6]; 7) Jischa-Rieke [7]; 8) Malhotra-Kang [5].

Of little benefit are the calculations by the k-ε model when "fitting" of the experimental data to the averaged heat transfer characteristics is achieved by suitable selection of Pr_T. In this case, depending on the form of approximation of separate terms in the balance equations for turbulent energy (k) and its dissipation intensity (ε), absolutely different Pr_T may be obtained, as shown by the data of [5] and [6] in Fig. 4 on the flow section-averaged turbulent Prandtl number at Pr > 1.

From the aforesaid it follows that in the absence of basic reliable data on turbulent heat transfer under standard conditions without the influence of additional factors on the near-wall turbulent flow (an equilibrium boundary layer, a stabilized channel flow, without angular zones) one cannot obtain verified experimental data and physically justified mathematical models for turbulent heat transfer involving the action of different factors. The question naturally arises about the structure of turbulent heat transfer. Use even of the simplest transfer characteristic in the form of Pr_T requires a definitive answer to a number of questions, in particular, about the dependence on the Pr and Re numbers in all practically necessary ranges; about the variation of the given Pr and Re numbers throughout the entire flow thickness provided dynamic and thermal structural characteristics are known for the considered conditions; about the boundary conditions on (near) a wall and at the external boundary.

The available experimental data testify to the fact that at relatively low Reynolds numbers ($Re < 2\cdot10^4$) a strong dependence of Pr_T on Re is observed (see Fig. 5) and its form essentially differs from that shown in Figs. 6 and 1. For rigorous determination of Pr_t variation in the vicinity of a wall it is necessary to take into account the propagation of temperature waves from the turbulent nonisothermal region to the wall (see Fig. 7). The influence of penetration of temperature fluctuations into the wall on the thermal fluctuation characteristics in the flow is determined by the ratio of heat accumulation by the corresponding media, i.e., the parameter $\Lambda=[(\rho c\lambda)_w/(\rho c_p\lambda)_f]^{1/2}$. In conformity with the estimates given in [16] at $\Lambda > 10^3$ temperature fluctuations on the wall surface may be considered equal to the zero, i.e., $t_w = 0$, while at $\Lambda < 0.1$ the heat flux density has no the fluctuations on the surface, i.e., $q'_w=-\lambda_w(\partial t/\partial y)_w=0$.

When the thermal boundary layer is smaller than the dynamic one (see the Figure in Table 1), one may observe temperature intermittence which is characterized by an abrupt change of the intensity of temperature fluctuations and extremal values of their excess and asymmetry at completely developed dynamic turbulence. Temperature intermittence characteristics depend, obviously, on the position of the boundary of the thermal boundary layer with respect to the depth of the dynamic boundary layer. Temperature intermittence in the case of a



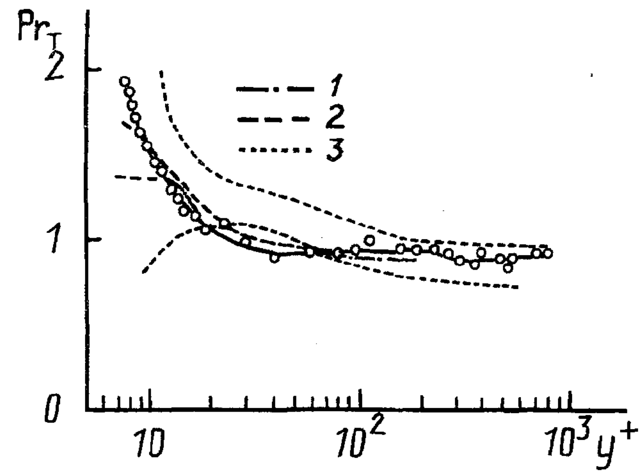


Fig. 5. Turbulent Prandtl number for an air flow in a tube: 1) Re = 5100; 2) 10^4; 3) $2 \cdot 10^4$.

Fig. 6. Turbulent Prandtl number versus universal coordinate in tubes for Pr = 0.7 [9]: dots, measurement results [9]; line [10] based on the dependence $Pr_t = \{1/2Pr_{T\infty} + (cPr\nu_T/\nu)/Pr_{T\infty}^{1/2} - (cPr\nu_T/\nu)^2\ [1 - \exp(-1/(cPr\nu_T Pr_{T\infty}^{1/2})/\nu))\ ]\}^{-1}$, $Pr_T = 0.86$; c = 0.2; 1) Kays-Grawford [10]; 2) Wassel-Catton; 3) Simpson et al. (uncertainty envelope).

steady-state turbulent air flow in a flat channel with one-sided heating [17] is shown in Fig. 8. With such a strong change of temperature fluctuations it is natural to expect also a considerable Pr_T change in the temperature intermittence zone; however, no results of turbulent heat transfer measurements under these conditions are known.

Recently several experimental studies of turbulent heat transfer components have been conducted [18-20]. Each of these works is concerned with a certain specific feature of turbulent heat transfer in a boundary layer. In [18] data are reported on tangential turbulent heat transfer at a specially created constant gradient of an averaged temperature across a flow and a plate in a fluid streaming parallel to its surface. The correlations $\langle wt \rangle$, $\langle vt \rangle$ directly measured by a hot-wire anemometer are given in gradient form as

$$\langle wt \rangle = a_{Tz} \frac{\partial T}{\partial z}, \quad \langle vt \rangle = a_{Ty} \frac{\partial T}{\partial y}, \tag{1}$$

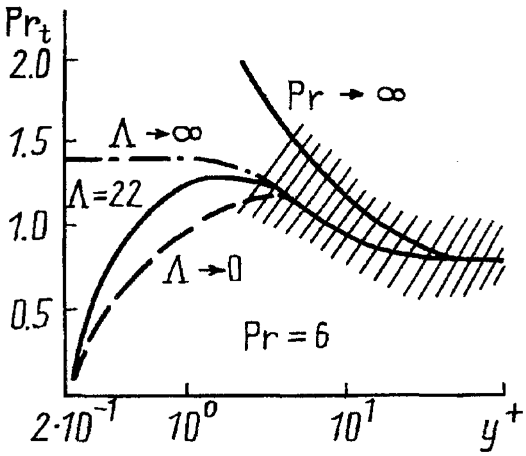
and the validity is discussed of the relation proposed by Londer

$$a_{Tz}/a_{Ty} = \langle w^2 \rangle / \langle v^2 \rangle . \tag{2}$$

As the results of [18] show, at uniform and moderate temperature gradients in the internal turbulent region of the boundary layer this relation is satisfied. However in the case of strong temperature variation with respect to z, in particular, on "strip-like" heating of a flow past a surface one would expect the effects of temperature intermettence (Fig. 8). The turbulent heat transfer in thermal boundary regions needs special studies.

The data on tangential $\langle wt \rangle$ and normal $\langle vt \rangle$ heat transfer are also discussed in [19]. However, whereas in [18] the tangential transfer is caused by thermal conditions, in [19] it is due to hydrodynamic conditions, namely, interaction of a mechanically induced longitudinal vortex with a thermal boundary layer. It is found that this interaction enhances turbulent heat transfer to a greater extent, as compared to momentum transfer enhancement, especially near a vortex core. The vector of the turbulent heat transfer in a flow cross section formed by the components $\langle vt \rangle$, $\langle wt \rangle$ is perpendicular, in fact, at all points to the isotherms, thus confirming the validity of the isotropic-diffusional-eddy model for the given conditions.

In the general case, the turbulent thermal diffusivity relating the turbulent heat transfer vector with the temperature gradient is a second-order tensor, i.e.,



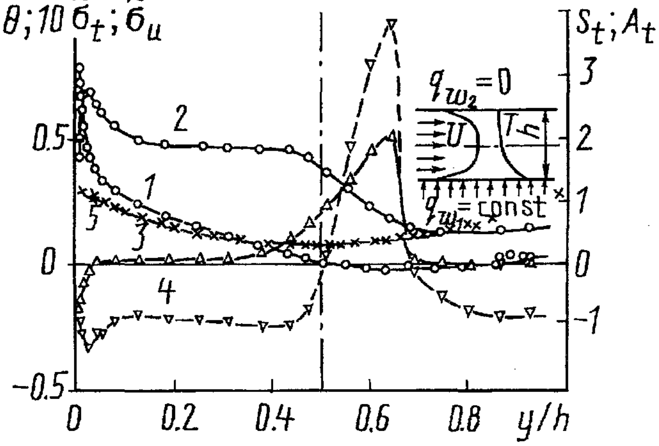


Fig. 7. Influence of the wall material on turbulent heat transfer in a viscous sublayer [16]; $\Lambda = 22$ for a flow around a stainless steel wall.

Fig. 8. Distribution of temperature field characteristics with respect to the channel height (Re = 32000): 1) $\theta = (T - T_0)/(T_w - T_0)$; 2) $\tilde{\sigma}_t = \sigma_t/T_*$; 3) S_t; 4) A_t; 5) $\tilde{\sigma}_u = \sigma_u/U = \langle u^2 \rangle^{1/2}/U$.

$$\langle u_i t \rangle = D_{ij} \frac{\partial T}{\partial x_j}. \tag{3}$$

In the simplest case, assuming $D_{12} = D_{21} = D_{13} = D_{31} = D_{23} = D_{32} = 0$ and $D_{11} = D_{22} = D_{33} = a_T$, neglecting heat transfer along the flow for the considered conditions, and using a scalar coefficient, we arrive at the following expression for transverse heat transfer:

$$\langle vt \rangle \mathbf{j} + \langle wt \rangle \mathbf{k} = a_T \left(\frac{\partial T}{\partial y} \mathbf{j} + \frac{\partial T}{\partial z} \mathbf{k} \right). \tag{4}$$

Provided the correlation components and the averaged temperature gradient are found experimentally, the scalar turbulent thermal diffusivity may be determined from the relation

$$a_T = \frac{[\langle vt \rangle^2 + \langle wt \rangle^2]^{1/2}}{\left[\left(\frac{\partial T}{\partial y} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right]^{1/2}}. \tag{5}$$

In the general case the eddy viscosity coefficient is a fourth (simplified second)-order tensor. If viscosity and thermal diffisivity are expressed in the form of second-order tensors (D_{kj}, E_{ij}), then the tensor of the turbulent Prandtl number will be determined by the expression

$$E_{ij} = (\mathrm{Pr}_T)_{ik} D_{kj}. \tag{6}$$

Physical-mathematical modeling of turbulent heat transfer with use of $(\mathrm{Pr}_T)_{ik}$ and especially the elaboration of practical calculation methods do not seem promising.

Assuming viscosity in the form of a diagonal tensor and neglecting longitudinal transfer for the case considered in [19], the relationship between the components of turbulent momentum transfer and the gradients of averaged velocity components may be written in the form

$$\langle uv \rangle = E_{22} \frac{\partial U}{\partial y}, \quad \langle uw \rangle = E_{33} \frac{\partial U}{\partial z}. \tag{7}$$

In this case the tensor of the turbulent Prandtl number is reduced to two values

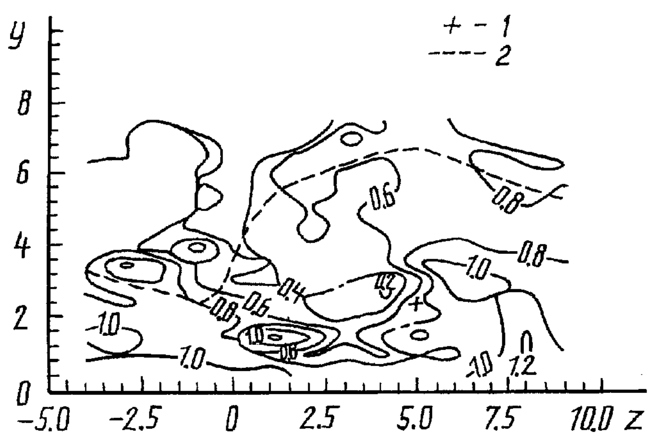


Fig. 9. Turbulent Prandtl number [19]: 1) vortex core; 2) $\bar{u}/u_\infty = 0.99$. y, z, cm.

$$(\mathrm{Pr_T})_{22} = E_{22}/D_{22}, \quad (\mathrm{Pr_T})_{33} = E_{33}/D_{33}. \tag{8}$$

Within the framework of the isotropic eddy viscosity approximation, whose validity with respect to the boundary layer thickness is limited, its scalar magnitude $\nu_T = E_{22} = E_{33}$ enters the definition of the scalar turbulent Prandtl number $\mathrm{Pr_T} = \nu_T/a_T$. Figure 9 presents experimental $\mathrm{Pr_T}$ data [19] for a flow cross section obtained in the described manner with use of expression (5) for a_T. In the flow near the vortex core the heat transfer in the z-direction and low $\mathrm{Pr_T}$ values (up to 0.2) prevail. Within the regions of the flow where the boundary layer may be assumed to preserve its two-dimensional character, $\mathrm{Pr_T} \approx 0.8{-}1$.

In [20] large-scale organized motions in the course of momentum and heat transfer in a developed turbulent boundary layer on a rough slightly heated surface are investigated. Measurements are made by recording the propagation of temperature nonuniformities. It is found that the contribution of large scale motions to heat transfer <vt> and <ut> makes up 35-45%, which is considerably higher than that for a smooth surface determined earlier experimentally on the same experimental setup. Their contribution to the momentum transfer is approximately of the same value, 35-40%; however it is higher by a factor of two than on a smooth plate.

To sum up, we draw the following conclusions.

1. Locally there are essential differences in turbulent momentum and heat transfer in a boundary layer characterized by the correlations $\langle u_i u_j \rangle$, $\langle u_i t \rangle$. Such a difference is even more pronounced for the structural characteristics of these transfer modes, which becomes even more essential when different factors exert an influence on a flow.

2. The notions of coefficients of turbulent momentum and heat (ν_T, a_T) transfer and a turbulent Prandtl number $\mathrm{Pr_T}$ are justified only within the framework of their scalar representation for the conditions of a two-dimensional boundary layer (see Table 1). The efforts to assign a tensor meaning to them in the case of three-dimensional flows lead to additional uncertainties in constructing physicomathematical models. Within the framework of the Reynolds approximations it seems necessary to leave any attempts to construct models for three-dimensional and nonstationary nonuniform anisotropic conditions with use of the analogs of molecular transfer, i.e., the quantities ν_T, a_T, $\mathrm{Pr_T}$. In connection with this, further elaboration of the k-ε model holds no promise. Calculation models must be based on direct modeling of the correlations $\langle u_i u_j \rangle$, $\langle u_i t \rangle$ or on abandoning the Reynolds approach but the latter requirement is the subject matter of another work.

3. To provide physical grounds for modeling the turbulent heat transfer in near-wall flows, it is necessary, first of all, to create a data bank of reliable experimental data on the characteristics and structures of propagating thermal disturbances in standard, undisturbed turbulent flows, which are better accomplished in a long smooth tube with a round cross section. It will become the first stage in the program of creating a reference data bank for turbulent heat transfer in shear flows.

The results of [21] may be considered as a certain contribution to settling the problem formulated in item 3.

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